PRACTICAL 6 SOLUTIONS

Q1. Let $X_1, X_2, \ldots, X_{10}$ denote Christmas spends for Aberystwyth residents.
(Two-sample t-test) Let $Y_1, Y_2, \ldots, Y_7$ denote Christmas spends for Machynlleth residents.
(units : £)

Assume $X_i \sim N(\mu_1, \sigma_1^2)$ independently for $i = 1, \ldots, 10$ and
$Y_i \sim N(\mu_2, \sigma_2^2)$ independently for $i = 1, \ldots, 7$.

Note that the sample standard deviations for Aber and Mach are respectively $\sqrt{1104 \cdot 5} = £33 \cdot 23$ and $\sqrt{6161 \cdot 2} = £78 \cdot 49$.

We will not assume equal variances.

We test $H_0 : \mu_1 = \mu_2$ vs
$H_1 : \mu_1 > \mu_2$.

here we're used independence of X_i, Y_i

We calculate $ESE(\bar{X} - \bar{Y})$: Since $Var(\bar{X} - \bar{Y}) = Var(\bar{X}) + Var(\bar{Y})$

$$= \frac{\sigma_1^2}{10} + \frac{\sigma_2^2}{7},$$

we have that $ESE(\bar{X} - \bar{Y}) = \sqrt{\frac{S_1^2}{10} + \frac{S_2^2}{7}} = \sqrt{\frac{1104 \cdot 494}{10} + \frac{6161 \cdot 2}{7}} = £31 \cdot 4741$

The t-statistic $T = \dfrac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{ESE(\bar{X} - \bar{Y})}$ is distributed as $t_{[\nu]}$, where

$$\nu = \frac{\left(\frac{S_1^2}{10} + \frac{S_2^2}{7}\right)^2}{\frac{1}{9}\left(\frac{S_1^2}{10}\right)^2 + \frac{1}{6}\left(\frac{S_2^2}{7}\right)^2} = 7 \cdot 52,$$

so we round down to 7 degrees of freedom.

(Here we have used the Welch-Aspin approximation)

Under the assumption of H_0, $T_{obs} = \dfrac{(153 \cdot 08 - 133 \cdot 69) - 0}{31 \cdot 4741} = 0 \cdot 616$.

From tables, $P(t_{[7]} > 0 \cdot 616) > 0 \cdot 25$. The test is not significant at the 25% level; insufficient evidence to reject H_0. That is, we do <u>not</u> reject the notion that Aberystwyth residents spend the same as Machynlleth residents.

Binomial question

Q1. Assuming Aberystwyth townsfolk are either "naughty" or "nice", independently with fixed probability p, we consider the number of "nice" people, denoted R say, in a sample of size n. Then $R \sim \mathrm{Bin}(n, p)$

In the below, we test $H_0: p = 0.8$

vs $H_1: p < 0.8$.

(i) $n = 10$, observe $r = 7$.

"At most 7 out of 10 are nice" is the same event as "at least 3 out of 10 are naughty"

$$p\bar{o} = P(R \leq 7) = P(\mathrm{Bin}(10, 0.8) \leq 7) = P(\mathrm{Bin}(10, 0.2) \geq 3)$$
$$= \underline{\underline{0.3222}} \quad \text{(from tables)}.$$

So $p\bar{o} > 0.1$, not significant at 10% level; insufficient evidence to reject H_0.

(ii) $n = 20$, observe $r = 14$.

$$p\bar{o} = P(R \leq 14) = P(\mathrm{Bin}(20, 0.8) \leq 14) = P(\mathrm{Bin}(20, 0.2) \geq 6)$$
$$= \underline{\underline{0.1958}} \quad \text{(from tables)}.$$

So $p\bar{o} > 0.1$, not significant at 10% level; insufficient evidence reject H_0.

(iii) $n = 140$, observe $r = 98$.

$$p\bar{o} = P(R \leq 98) = P(\mathrm{Bin}(140, 0.8) \leq 98) = P(\mathrm{Bin}(140, 0.8) < 98.5)$$
$$\doteqdot P\left(Z < \frac{98.5 - 140 \times 0.8}{\sqrt{140 \times 0.8 \times 0.2}}\right)$$

NORMAL APPROX. AND CONTINUITY CORRECTION

$$= P(Z < -2.8524)$$
$$= P(Z > +2.8524) \quad \text{(by symmetry of Normal distribution)}$$
$$= 0.0022,$$

where $Z \sim N(0, 1)$. Since $p\bar{o} < 0.01$, the test is significant at the 1% level, so we have strong evidence to reject H_0.