# Practical 5: One Normal Sample.

**Q1.** (a) (i) $P(Z > 2.08) = \underline{0.0188}$ (directly from tables)

(ii) $P(Z < 0.19) = 1 - P(Z > 0.19) = 1 - 0.4247 = \underline{\underline{0.5753}}$

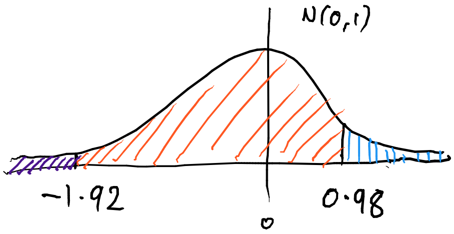
(b) If $X \sim N(55, 6^2)$, then $Z = \dfrac{X - 55}{6} \sim N(0,1)$.

(i) $P(X < 69.82) = P\left(Z < \dfrac{69.82 - 55}{6}\right) = P(Z < 2.47) = 1 - P(Z > 2.47)$

$= 1 - 0.0068$

$= \underline{\underline{0.9932}}$

(ii) $P(43.48 < X < 60.88) = P\left(\dfrac{43.48 - 55}{6} < Z < \dfrac{60.88 - 55}{6}\right)$




$= P(-1.92 < Z < 0.98)$

$= 1 - P(Z > 0.98) - P(Z < -1.92)$

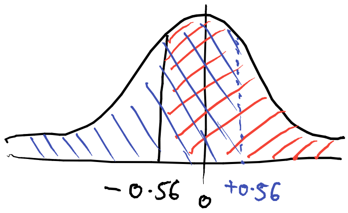
$= 1 - 0.1635 - P(Z > 1.92)$ (by symmetry)

$= 1 - 0.1635 - 0.0274$

$= \underline{\underline{0.8091}}$

(c) The sum of independent Normals is a Normal: $S \sim N(220, 144)$.

Thus $P(S > 213.28) = P\left(Z > \dfrac{213.28 - 220}{12}\right) = P(Z > -0.56)$, where $Z \sim N(0,1)$.




$= P(Z < 0.56)$ (by symmetry)

$= 1 - P(Z > 0.56)$

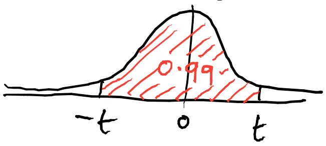
$= 1 - 0.2877 = \underline{0.7123}$

**Q2.** (a) $P(t_{[13]} > t) = 0.01 \Rightarrow \underline{t = 2.6503}$ (direct from tables)

(b) $P(t_{[20]} < -t) = P(t_{[20]} > t)$ (by symmetry)

$= 0.025 \Rightarrow \underline{\underline{t = 2.086}}$

(c) $P(-t < t_{[40]} < t) = 1 - 2P(t_{[40]} > t) = 0.99$




$\Rightarrow 2P(t_{[40]} > t) = 0.01$

$\Rightarrow P(t_{[40]} > t) = 0.005$

$\Rightarrow \underline{\underline{t = 2.7045}}$

Q3. Let $X$ denote the amount spent by a motorist at the petrol station, in pounds. Assuming $X \sim N(\mu, \sigma^2)$ independently for each motorist, then the 95% confidence interval for $\mu$ is given by:

$$\bar{X} \pm t_{0.025[9]} \times ESE(\bar{X}) = 58.30 \pm 2.2622 \times 5.25$$
$$= 58.30 \pm 11.87655$$
$$= (46.42,\ 70.18).$$

We conclude that, with 95% confidence, the average spend of a motorist at the petrol station lies between £46.42 and £70.18.

Q4. Let $X$ denote the concentration of lead compounds in a sample of river water. Assuming $X \sim N(\mu, \sigma^2)$ independently for each sample, then the 90% confidence interval for $\mu$ is given by:

$$\bar{X} \pm t_{0.05[5]} \times ESE(\bar{X}) = 0.6 \pm 2.0150 \times \frac{0.14}{\sqrt{6}}$$
$$= 0.6 \pm 0.1152$$
$$= (0.4848,\ 0.7152).$$

We conclude, with 90% confidence, that the average concentration of lead compounds per sample of river water lies between 0.48 and 0.72 units.

Q5. (a) From tables, $p_0^+$ lies between 0.25% and 0.5%. This is significant at the 0.5% level, thus giving strong evidence against $\mu = 0$ in favour of the alternate hypothesis $\mu > 0$.

(b) From tables, $p_0^- = P(t_{[11]} < -1.327) = P(t_{[11]} > 1.327)$ (by symmetry) lies between 10% and 15%. This is not significant at the 10% level; there is little evidence against $H_0: \mu = 10$.

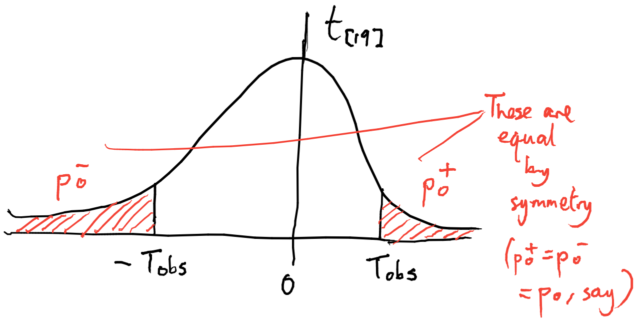
(c) From tables, $p_0^+ = P(t_{[13]} > 4.295)$ is less than 0.05%. The test is significant at the 0.05% level; there is very strong evidence against $H_0: \mu = 3$. We therefore reject $H_0$ in favour of $H_1: \mu > 3$.

(d) Under $H_0$, the t-statistic is $T_{obs} = \frac{\bar{X} - \mu_0}{ESE(\bar{X})} = \frac{2.5 - 2}{0.23}$, and $T \sim t_{[15]}$.
Thus $p_0^+ = P\left(t_{[15]} > \frac{2.5-2}{0.23}\right) = P(t_{[15]} > 2.1739)$, which lies between 1% and 2.5%. The test is therefore significant at the 2.5% level, so we have moderately strong evidence against $H_0: \mu = 2$.

(e) Under $H_0$, the t-statistic is $T_{obs} = \frac{\bar{X}-\mu_0}{ESE(\bar{X})} = \frac{\bar{X}-\mu_0}{S/\sqrt{n}} = \frac{-7.3}{26.23/\sqrt{30}} = -1.5243$, and $T \sim t_{[29]}$. Thus $p_0^- = P(t_{[29]} < -1.5243) = P(t_{[29]} > 1.5243)$ by symmetry. From tables we note that $p_0^-$ lies between 5% and 10%. This is a borderline case but evidence against $\mu = 0$ is not strong enough to reject $H_0: \mu = 0$.

(f) Under $H_0$, the t-statistic is $T_{obs} = \frac{\bar{X}-\mu_0}{ESE(\bar{X})} = \frac{-53}{30.8} = -1.7208$, and $T \sim t_{[19]}$. Thus $p_0^- = P(t_{[19]} < -1.7208) = P(t_{[19]} > 1.7208)$ by symmetry. From tables we note that $p_0^-$ lies between 5% and 10%, but is very close to 5%. This is a borderline case, but very nearly significant at the 5% level. It's therefore probably best to say there's some evidence against $\mu=0$, but not strong.

**Q6** In each of the following, under $H_0$, the t-statistic is distributed as $T \sim t_{[19]}$. The significance level is therefore $P(t_{[19]} > T_{obs}) + P(t_{[19]} < T_{obs})$
$= p_0^+ + p_0^- = 2p_0$ by symmetry.




(i) $p_0^+ = P(t_{[19]} > 2.13)$, which is between 1% and 2.5%. The significance level is therefore between 2% and 5%. Moderate evidence against $H_0$ $(\mu=0)$ in favour of $H_1$ $(\mu \neq 0)$.

(ii) $p_0^- = P(t_{[19]} < -1.52) = P(t_{[19]} > 1.52)$, which is between 5% and 10%. The significance level is therefore between 10% and 20%. Insufficient evidence to reject $H_0: \mu=0$.

(iii) $p_0^+ = P(t_{[19]} > 1.94)$, which is between 2.5% and 5%, so the significance level is between 5% and 10%. Insufficient evidence to reject $H_0: \mu=0$.

(iv) $p_0^+ = P(t_{[19]} > 2.09)$, which is between 2.5% and 5% (very close to 2.5%). Therefore the significance level is very close to 5%, although the test is not quite significant at the 5% level. There is therefore some evidence against $H_0: \mu=0$, but not very strong.

**Q7** Let $U_i$ denote the weight gain of the chick fed on the usual diet from brood $i$, and let $V_i$ denote " supplemented diet ".
We compute the differences $D_i = V_i - U_i$:

| BROOD (i) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $D_i$ | 3.12 | 1.84 | 0.56 | 3.66 | 3.06 | 0.06 | 2.53 | 1.32 |

We assume that each brood is independent from each other brood, but that the expected weights of chicks from a given brood on the same diet are equal. Thus $D_i \sim N(\mu, \sigma^2)$, independently for $i = 1, \ldots, 8$. Note that $\mu$ therefore represents the mean extra weight gained by a chick consuming the supplemented diet, as opposed to the usual diet.
We test $H_0: \mu = 0$ (the weight gains under the usual diet are the same as under the supplemented diet) against $H_1: \mu > 0$ (the weight gains are greater for the supplemented diet). Under $H_0$, the t-statistic is $T_{obs} = \frac{\bar{X}-\mu_0}{ESE(\bar{X})} = \frac{2.01875 - 0}{1.6767/\sqrt{8}} = 3.4054$, (since $\frac{1}{7}\sum_{i=1}^{8}(D_i - \bar{D})^2 = 1.6767$.)
and $T \sim t_{[7]}$.
Thus $p_0^+ = P(t_{[7]} > 3.4054)$, which from tables is between 0.5% and 1%. The test is significant at the 1% level; thus strong evidence against $H_0: \mu = 0$ in favour of $H_1: \mu > 0$. The supplement-eating chicks gain more weight than those on the usual diet.