MA26620: Practical 5

R detox: Confidence Intervals and Hypothesis Tests for Normal Samples

Good morning! I imagine that after the assignment, you might be glad to do some statistics that doesn't involve R. Consequently, today we'll tackle some pen-and-paper problems that use the content we've seen recently in the lectures (specifically the one-sample cases). Do ask for help if you need it. Also, the practical today is not too long; if you finish with plenty of time left, now may be a good time to write some revision notes on what you've done in R that you can refer to during next semester.

1 Using Statistical Tables

The following two questions provide practice/revision of the use of Normal and t tables. You can find a PDF of the Statistical Tables on the module webpages (see Stats Tables on the left menu).

Q1. Use Statistical Tables to evaluate the following:

(a) (i) $P(Z > 2.08)$, (ii) $P(Z < 0.19)$, if $Z \sim N(0, 1)$.

(b) (i) $P(X < 69.82)$, (ii) $P(43.48 < X < 60.88)$, if $X \sim N(55, 36)$.

(c) $P(S > 213.28)$, if $S = X_1 + X_2 + X_3 + X_4$, where X_1, X_2, X_3, X_4 are independent and $N(55, 36)$.

Q2. Use Statistical Tables to evaluate the value t such that

(a) $P(t_{[13]} > t) = 0.01$.

(b) $P(t_{[20]} < -t) = 0.025$.

(c) $P(-t < t_{[40]} < t) = 0.99$.

2 One Sample t Confidence Intervals

Recall from the lecture notes (or see the summary on the module webpages) that the following interval gives a $100(1 - 2\alpha)\%$ confidence interval for the true population mean μ:

$$\bar{X} \pm t_{\alpha[n-1]} \times ESE(\bar{X}).$$

That is, regardless of the true (but unknown) values of μ and of σ^2, $100(1 - 2\alpha)\%$ of samples' confidence intervals will contain μ. So this interval is called a $100(1 - 2\alpha)\%$ Confidence Interval for μ.

For a 95% confidence interval for instance, $100(1 - 2\alpha) = 95$, so we would choose $\alpha = 0.025$. This means that the t value we need to use in the confidence interval calculation would be found in the $(n - 1)^{\text{th}}$ row of the stats tables, and the column with "one-tailed" probability of 0.025. Recall that $S/\sqrt{n}$ is the estimated standard error of the sample mean.

Q3. A random sample of 10 motorists buying petrol are found to spend an average of £58.30 with estimated standard error £5.25. Calculate a 95% confidence interval for the expected spending of motorists at this petrol station.

Q4. Six samples of water are taken from a river and the concentration of lead compounds is measured. If the average concentration is 0.6 units with standard deviation 0.14, calculate a 90% confidence interval for the expected concentration.

3 One Sample t Hypothesis Tests

In each of the following t-tests, you may assume each t-statistic is calculated from a single Normal $N(\mu, \sigma^2)$ random sample.

- Use the appropriate row of the Statistical Tables page 12 to deduce a range of values within which the p-value, p_0, i.e. the observed significance level, lies;
- State whether the null hypothesis is rejected/not rejected and at what level, e.g. 5%, 1%, 0.1%, 10% etc.;
- Interpret p_0 by stating how strong the evidence is against H_0. You will find guidance on this on the module webpages under 'Hypothesis tests' (at the bottom of the page).

Q5. (a) Test $H_0 : \mu = 0$ vs $H_1 : \mu > 0$, when the t-statistic is 3.975 on 7 degrees of freedom.

(b) Test $H_0 : \mu = 10$ vs $H_1 : \mu < 10$, t-statistic $= -1.327$, sample size 12.

(c) Test $H_0 : \mu = 3$ vs $H_1 : \mu > 3$, t-statistic $= 4.295$, sample size 14.

(d) Test $H_0 : \mu = 2$ vs $H_1 : \mu > 2$, $n = 16$, $\bar{X} = 2.5$ with $ESE(\bar{X}) = 0.23$.

(e) Test $H_0 : \mu = 0$ vs $H_1 : \mu < 0$, where $n = 30$, $\bar{X} = -7.3$ and $S = 26.23$.

(f) Test $H_0 : \mu = 0$ vs $H_1 : \mu < 0$ where $n = 20$, $\bar{X} = -53$ and $ESE(\bar{X}) = 30.8$.

Q6. When testing against a two-sided alternative such as $H_1 : \mu \neq 0$, there are two directions in which we could erroneously reject H_0 when it is true. We allow for this by simply doubling the calculated p_0.

Carry out t-tests of the null hypothesis $H_0 : \mu = 0$ versus the alternative $H_1 : \mu \neq 0$, based on a random sample of size 20 from $N(\mu, \sigma^2)$, when the t-statistic takes the values:

(i) 2.13;
(ii) –1.52;
(iii) 1.94;
(iv) 2.09.

Q7. In a study of a dietary supplement for chickens, an investigator chose eight hens at random and, from each hen's brood of chicks, took two (again randomly selected). He then assigned the supplement to one chick out of each pair by the toss of a coin, the other chick receiving the usual diet. Their weight gains at the same standard age are given below.

Brood	1	2	3	4	5	6	7	8
Usual Diet	23.32	20.85	19.81	15.33	25.84	22.34	20.39	15.06
Supplement	26.44	22.69	20.37	18.99	28.90	22.40	22.92	16.38

Calculate the differences in weight gain. Carry out a t-test to assess the strength of evidence against the null hypothesis that the supplement is ineffective in increasing the weight gain. In writing this up make sure you describe (be clear but succinct!)

(i) your assumptions regarding the distribution of the data, i.e. the mathematical model; what does the mean μ refer to here? (ie the expected value of what?)
(ii) the hypotheses tested and what each implies about the diet;
(iii) the values of the t statistic and the p-value; how are they related?
(iv) whether H_0 is rejected and at what level;
(v) the strength of evidence and your conclusion about the supplement.